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Radiation by Uniformly Moving Sources

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Abstract

Uniformly and rectilinearly moving charges and other sources with zero eigenfrequency do not radiate in vacuo. In the case of motion in a medium, however, the absence of radiation is rather the exception. Radiation is absent if in a non-moving, homogeneous and time-independent medium a charge moves uniformly at a velocity v, smaller than the phase velocity c_{ph} for all waves that can propagate in this medium. If $v > c_{\text{ph}}$, the Vavilov–Cherenkov radiation may occur. However, radiation is also possible for $v < c_{\text{ph}}$. This is a transition radiation; it occurs if a source with a zero eigenfrequency moves at constant velocity in an inhomogeneous and (or) nonstationary medium (or near such a medium). The simplest form of transition radiation occurs when a charge crosses the boundary between two media. If the properties of a medium change periodically, the transition radiation is somewhat specific (resonance transition radiation or transition scattering). In particular, transition scattering occurs when a permittivity wave falls on a non-moving charge. Transition scattering is closely connected with transition bremsstrahlung radiation. Transition processes are essential for plasma physics.

We also touch upon the quantum interpretation of the Vavilov–Cherenkov effect and the classical and the quantum theory of the Doppler effect in a medium. The Vavilov–Cherenkov effect, transition radiation and transition scattering have analogies beyond the limits of electrodynamics to be described below.

1. Introduction

There exists a rather large section of electrodynamics—and, in particular, optics—which deals with the radiation of uniformly and rectilinearly moving sources and, first of all, charges. The effects to be discussed below have analogies beyond the limits of electrodynamics (acoustics, chromodynamics and, properly speaking, any field theory).

We shall deal with the radiation of a source moving uniformly in (or near) a medium. Most important are sources with zero eigenfrequency, i.e. static in the reference frame where they are at rest. The sources may be charges, various permanent dipoles, etc. The corresponding radiation is the Vavilov–Cherenkov * radiation or transition radiation in its different forms. If a source has a non-zero eigenfrequency and moves uniformly and rectilinearly, we can observe the Doppler effect. For motion in vacuo the Doppler effect is well-known, but in a medium it may be more complicated (in textbooks this case is usually not considered).
We have chosen the radiation of uniformly moving charges as the subject of the present chapter for three reasons. First, the corresponding class of problems seems to be rather interesting both from the general physical point of view and for the analysis of a number of concrete effects and their application. Second, I have been engaged in the study of these problems from the very beginning of my scientific activities. Third, there exists a fairly close connection with the studies of Niels Bohr who published his famous papers [1,2] devoted to the motion of charged particles through matter, in 1913 and 1915.

A consistent application of electrodynamics of continuous media to the motion of fast charged particles through matter is presented in a book by Landau and Lifshitz [10, ch. 14] (note, however, that for different dipoles and higher multipoles the situation is more complicated than for charges; this question is considered below, in section 5). For charges the role of the medium is essential only for far collisions with an impact parameter $\rho \gg a$, where for a condensed medium $a \sim 3 \times 10^{-10}$ m is the atomic dimension and for a plasma $a \sim N^{-1/3}$ is the mean distance between particles. Under such conditions, say, a radiation of waves with a wavelength $\lambda \gg a$ can be considered on the basis of the equations of electrodynamics of continuous media. Assuming for simplicity the medium to be isotropic and neglecting spatial dispersion, we can take into account all the properties of the medium by introducing the dielectric constant $\epsilon(\omega)$ and the magnetic permeability $\mu(\omega)$.

So, we shall deal with a transparent medium with a refractive index $n(\omega) = \sqrt{\epsilon(\omega)\mu(\omega)}$ and disregard absorption. In the case of transition radiation, however, we also consider an absorbing or nontransparent medium (for a nonabsorbing but nontransparent medium ϵ and μ are real quantities, but $\epsilon\mu < 0$). Here we only present the most important results and make some remarks. References may be found in some original papers [3–5,7,9] and books [8,11,12].

* In the Western literature this effect is called the Cherenkov radiation. We (my colleagues, acquainted with the history of the question, and the author; see [6]) believe, however, that only the term "Vavilov–Cherenkov effect" is correct.

2. Vavilov–Cherenkov effect for a charge

If a charge moves uniformly, not in vacuo, but in a medium, the absence of radiation is the exception rather than the rule. In fact, a charge emits no radiation only if its velocity v is less than the phase velocity c_{ph} for all of the electromagnetic waves which can propagate in a given medium. If $v > c_{ph}$, the Vavilov–Cherenkov radiation occurs. Moreover, even if $v < c_{ph}$ the absence of radiation refers only to a medium at rest, which is everywhere uniform and, besides, time-independent. In a non-uniform and/or non-stationary medium a uniformly moving charge radiates also if $v < c_{ph}$: this is the transition radiation. It is interesting that even a charge at rest can radiate in a medium, although this is a rather exotic case (see section 8).

Sommerfeld considered in some detail the following problem: a charge (charged pellet) moves in vacuo at a velocity $\boldsymbol{v} =$ constant and we must find its electromagnetic field. For $v < c$ there is no radiation (a stationary problem is considered). For $v > c$ there occurs radiation and its front forms a conic surface with an angle θ_0 between the normal to it (the wavevector $\boldsymbol{k}$) and the velocity $\boldsymbol{v}$. In this case (see fig. 1):

$$\cos\theta_0 = \frac{c}{v}. \tag{1}$$

One may say that condition (1) has a kinematic character—it is the condition for interference of secondary waves excited by the charge along its path. Rewritten in the form

$$\cos\theta_0 = \frac{c_{ph}}{v}, \tag{2}$$

condition (1) holds for any kind of wave and for any static source; but it can be satisfied only if

$$v > c_{ph}. \tag{3}$$

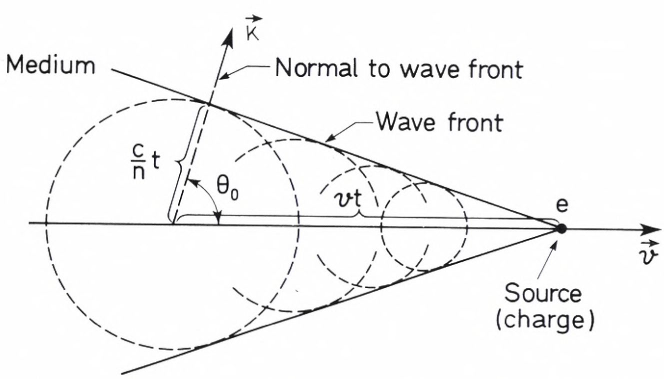

Fig. 1. Formation of Vavilov–Cherenkov radiation; $(c/n)t$ is the light path for the time t, vt is the length passed by the particle during the same time.

Sommerfeld had in a sense bad luck: within a year, we write 1905, the special theory of relativity appeared. According to this theory, the particle momentum is given by

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}}$$

and it is impossible to accelerate it from small velocities to a velocity $v > c$. Moreover, it would seem that the causality requirements also do not allow particles to move with a velocity $v > c$, since they could be used as superluminal signals.

Nobody thought of the simple idea of transferring Sommerfeld's results to the case of a charge moving through a medium. True, even before the appearance of Sommerfeld's papers Heaviside [13] understood correctly the possible role of the medium. However, at that time his papers did not attract due attention. History took a different run. In 1934. S.I. Vavilov and P.A. Cherenkov observed radiation, the nature of which was explained in 1937 by Frank and Tamm [4] (for the history of the discovery see ref. [6]).

Vavilov–Cherenkov radiation is the radiation of a uniformly moving charge in a transparent medium with refractive index $n(\omega)$ and, hence, with phase velocity $c_{\mathrm{ph}} = c/n(\omega)$. Conditions (2), (3) thus become

$$\cos\theta_0 = \frac{c}{n(\omega)\, v}, \tag{4}$$

and

$$v \geqslant \frac{c}{n(\omega)}. \tag{5}$$

Frank and Tamm obtained for the energy emitted by a particle of charge e per unit time (that is, along a pathlength v) the following expression:

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \frac{e^2 v}{c^2} \int_{\frac{c}{n(\omega)\, v} \leqslant 1} \mu(\omega) \left(1 - \frac{c^2}{n^2(\omega)\, v^2}\right) \omega \, \mathrm{d}\omega$$

$$= \frac{e^2 v}{c^2} \int \mu(\omega)\left[\sin^2\theta_0(\omega)\right] \omega \, \mathrm{d}\omega. \tag{6}$$

Vavilov–Cherenkov radiation now occupies a prominent place in physics, and a huge number of papers is devoted to it, including books and review articles (see refs [8,11,12] and the literature cited therein). Not a smaller role is played by the Landau damping, i.e. the damping of longitudinal (plasma) waves in a collisionless plasma. Landau [14] concluded that such a damping should exist when, in 1946, he was working on the solution of the initial value problem concerning the propagation of longitudinal perturbations in a collisionless plasma, using the kinetic equation. The collisionless damping which then occurs and which also appears in a number of problems of plasma physics and plasmalike media (for instance, in the case of the "solid-state plasma"—the electron liquid in metals), can be interpreted (if one considers it from a physical point of view) in different ways. One of them is the following: the condition for collisionless wave absorption by electrons in the plasma, which has the form:

$$\omega = \boldsymbol{k}\boldsymbol{v}, \tag{7}$$

is simply the condition for emission (2) for waves (in this case the longitudinal plasma waves) with a phase velocity

$$c_{\mathrm{ph}} = \frac{\omega}{k} = \frac{c}{n_\ell}, \qquad n_\ell(\omega) = \frac{ck}{\omega}, \tag{8}$$

where $n_\ell(\omega)$ is the refractive index for the longitudinal waves considered. The collisionless Landau absorption is thus closely connected with the inverse Vavilov–Cherenkov effect for plasma waves (if recoil is neglected, the kinematic conditions for absorption and for emission of waves are the same). When we are dealing with an "external" wave (in this case a longitudinal wave propagating in the plasma) we must consider its interaction not only with a single particle, but with an ensemble of them. As a result it is necessary to take into account not only the absorption of the waves, but also the stimulated emission.

Up to recently it seemed rather obvious that Vavilov–Cherenkov radiation would be impossible in vacuo and also in media with a refractive index $\mathrm{n}(\omega) < 1$ (in particular, in an isotropic plasma under conditions where the well-known formula $n(\omega) = \sqrt{1 - \omega_{\mathrm{p}}^2/\omega^2}$ is valid). In fact, however, such a conclusion is incorrect or too rash (see [8, ch. 8]). Quite realistic radiation sources can move at a velocity $v > c$ (particle beams incident upon a metal plate), and there appears a possibility to observe the Vavilov–Cherenkov effect both in vacuo (under normal conditions, however, only if a boundary is present) and also in an isotropic plasma.

The Vavilov–Cherenkov effect is possible in vacuo also far from any boundaries if a strong constant magnetic field $\boldsymbol{B}$ exists which is comparable with the well-known critical value $B_{\mathrm{c}} = m^2c^3/e\hbar = 4.4 \times 10^{13}$ Gauss (here e and m are the charge and the mass of the electron, respectively). As was noted already in the beginning of the 1930s, vacuum in a strong field behaves like a bi-refringent medium. In some cases the refractive index for weak electromagnetic waves propagating in a strong magnetic field is $n_{\mathrm{i}} > 1$ and, hence, a uniformly moving charged particle can emit Vavilov–Cherenkov waves. Here we should not be confused by the fact that (unless we consider motion strictly in the direction of the strong magnetic field) a charged particle can be deflected by the magnetic field. The fact is that we can, in principle, maintain a constant velocity $\boldsymbol{v}$ of a particle by some external means (sources). Besides, one can formally assume the particle mass m to be arbitrarily large; its velocity will then be constant.

I wish to make yet one more methodical remark.

Frank and Tamm derived eq. (6) in 1937 by evaluating the electromagnetic energy flux S through a cylindrical surface surrounding the trajectory of the particle. The present author obtained the same formula in 1939 by evaluating the change in energy $dW_{e\ell m}/dt$ of the electromagnetic field per unit time in the entire space (see [15] and [8, ch. 1]). Finally, the same result (see [9,10]) can be obtained by evaluating the work done by the field on the particle per unit time, i.e. the quantity $e\boldsymbol{v}\boldsymbol{E}$, where the field $\boldsymbol{E}$ is calculated at the position of the charge ($e\boldsymbol{E}$ is the radiative friction force; the other parts of the field do not contribute to the corresponding expressions).

One would perhaps expect that all three methods would give identical results. Indeed, this is the case with the Vavilov–Cherenkov effect. But in the general case for non-stationary charges in vacuo (and in a medium) and, for example, for transition radiation, the quantities S, $dW_{e\ell m}/dt$ and $e\boldsymbol{v}\boldsymbol{E}$ may be different (for details see refs [8,12]).

3. The quantum theory of the Vavilov–Cherenkov effect

Let us now turn to the quantum interpretation of the Vavilov–Cherenkov effect. In general, the Vavilov–Cherenkov effect is described in the framework of the classical theory and quantum corrections are not important. It seems to me, however, that from a methodical (and, if you like from a physical) point of view the quantum approach is useful and interesting.

Let us restrict ourselves to obtain condition (4) for emission and to its quantum generalization (of course, quantum theory enables us to derive eq. (6) with its quantum corrections [16]).

How can one explain, in quantum-mechanical language, the absence of emission by a charge or another static source moving uniformly in vacuo? To do this, it is sufficient to use the energy and momentum conservation laws:

$$E_0 = E_1 + \hbar\omega, \qquad E_{0,1} = \sqrt{m^2c^4 + c^2p_{0,1}^2}\,, \tag{9}$$

$$\boldsymbol{p}_0 = \boldsymbol{p}_1 + \hbar\boldsymbol{k}, \qquad \hbar k = \frac{\hbar\omega}{c}, \tag{10}$$

where $E_{0,1}$ and $\boldsymbol{p}_{0,1}$ are the energy and the momentum of the charge (source) of rest mass m before (index 0) and after (index 1) the emission of a photon of energy $\hbar\omega$ and momentum $\hbar\boldsymbol{k} = (\hbar\omega/c)(\boldsymbol{k}/k)$ (ω is the radiation frequency). One can verify that it is impossible (and this is also clear from eq. (13) with $n = 1$) to satisfy relations (9) and (10) for $\omega > 0$.

In order to consider radiation by a source in a medium, one must know the energy and the momentum of the radiation (the expression for the energy $E = \sqrt{m^2c^4 + c^2p^2}$ of the source is, evidently, not changed). It is not that simple to do this fully consistently, but on an intuitive level the answer is clear at once. Indeed, the presence of the non-moving and time-independent medium does not affect the frequency ω at all, and the wavelength in the medium $\lambda = \lambda_0/n(\lambda)$, where $\lambda_0 = 2\pi c/\omega$ is the wavelength in vacuo; in other words, in the medium the wavenumber is

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c}n(\omega).$$

Using this substitution, we get, instead of eq. (10),

$$\boldsymbol{p}_0 = \boldsymbol{p}_1 + \hbar\boldsymbol{k}, \qquad k = \frac{\hbar\omega\, n(\omega)}{c}, \qquad \boldsymbol{p}_{0,1} = \frac{m\boldsymbol{v}_{0,1}}{\sqrt{1 - v_{0,1}^2/c^2}}. \tag{11}$$

A simultaneous solution of eqs. (9) and (11) leads to the result

$$\cos\theta_0 = \frac{c}{n(\omega)v_0}\left(1 + \frac{\hbar\omega(n^2-1)}{2mc^2}\sqrt{1 - \frac{v_0^2}{c^2}}\right), \tag{12}$$

$$\hbar\omega = \frac{2(mc/n)(v_0\cos\theta_0 - c/n)}{\sqrt{1 - v_0^2/c^2}\,(1 - 1/n^2)}, \tag{13}$$

where θ_0 is the angle between $\boldsymbol{v}_0$ and $\boldsymbol{k}$. If

$$\frac{\hbar\omega}{mc^2} \ll 1, \tag{14}$$

[or for a somewhat more general inequality resulting from eq. (12)], which corresponds to the classical limit, eq. (12) changes to eq. (4), as one expects. The classical limit corresponds to neglecting the recoil when a "photon in the medium" with momentum $\hbar\boldsymbol{k}$ is emitted. It is also clear from eq. (13) that $\omega > 0$ and $\cos\theta_0 < 1$, i.e. emission is possible, only for $v_0 > c/n(\omega)$ [see eq. (5)]. In the classical limit, where the result [in this case expression (4)] does not contain Planck's constant $\hbar$, the quantum calculation has merely a methodical character: it may turn out to be convenient, but it is not unavoidable. The energy and momentum conservation laws can be formulated in the classical region by taking into account the connection between the emitted energy $W_{e\ell m}$ and the change in momentum $\boldsymbol{G}$ of the radiation and of the medium. In accordance with eq. (11) we get

$$\boldsymbol{G} = \frac{W_{e\ell m}n}{c}\cdot\frac{\boldsymbol{k}}{k}. \tag{15}$$

Further, for a freely moving particle with sufficiently small changes in energy and momentum,

$$\Delta E - E_1 - E_0 = \boldsymbol{v}\Delta\boldsymbol{p} = \boldsymbol{v}(\boldsymbol{p}_1 - \boldsymbol{p}_0);$$

indeed,

$$\frac{\mathrm{d}E}{\mathrm{d}\boldsymbol{p}} = \frac{\mathrm{d}}{\mathrm{d}\boldsymbol{p}}\sqrt{m^2c^4 + c^2p^2} = \frac{c^2\boldsymbol{p}}{E} = \boldsymbol{v},$$

furthermore one can put $v_0 \approx v_1 \approx v$, and from the conservation laws (9) and (11), and replacing $\hbar\omega$ by $W_{e\ell m}$, we obtain

$$\Delta E = W_{e\ell m} = \boldsymbol{v}\Delta \boldsymbol{p} = \frac{W_{e\ell m} n}{c}\left(\frac{\boldsymbol{k}\boldsymbol{v}}{k}\right).$$

The energies $W_{e\ell m}$ cancel out, and we are thus led to the classical condition for emission [see eq. (4)]

$$\frac{nv}{c}\cos\theta_0 = 1$$

Relation (15), or $k = \hbar\omega n/c$ [see eq. (11)] corresponds to writing the energy-momentum tensor in a medium in the Minkowskii form. In fact, however, the energy-momentum tensor of the field in a medium has the form proposed by Abraham (we mean the simplest case of a non-dispersive medium) which is reflected in the existence of the Abraham force upon the medium with a density

$$\boldsymbol{f}^{\mathrm{A}} = \frac{n^2 - 1}{4\pi c}\frac{\partial}{\partial t}[\boldsymbol{E}\boldsymbol{H}].$$

One can show that expression (15) is valid also when we use the Abraham tensor, if we are interested in the total momentum of both radiation and medium (for details see ref. [8], ch. 12; the connection with the phonon momentum in a solid body is also treated there). But it is just that quantity which occurs in the conservation law (11) or its classical analogue. The necessity to use expressions (11) and (15) for $\hbar\boldsymbol{k}$ and $\boldsymbol{G}$ follows also from the classical eq. (4) for the angle of the Vavilov–Cherenkov radiation.

4. *Vavilov–Cherenkov radiation in the case of motion in channels and gaps*

Energy losses due to Vavilov–Cherenkov radiation make up a part of the total, so-called ionization losses. Vavilov–Cherenkov radiation in a transparent medium can go far from the source trajectory. The possibility to eliminate practically all other losses seems nonetheless very interesting. To this end a radiating particle must move in an empty channel or gap in a medium. In this case the losses due to near collisions are absent altogether, polarization losses are strongly suppressed (see also [17]), and the Vavilov–Cherenkov radiation on waves with wavelength λ changes little if $\lambda \gg r$, where r is the radius of the channel, or the width of the gap. This fact was mentioned by L.I. Mandelstam in 1940 (when he spoke at Cherenkov's thesis defence). The transverse electromagnetic field of a charge in a direction perpendicular to the source trajectory is formed in a region on the order of λ. If $r \gg a$, the influence of a channel or a gap can be accounted for by macroscopic equations with the usual boundary conditions. Such calculations [18] show, for example, that in a medium with $n = 1.5$ a charge with $v \to c$ in an empty channel, for which $r/\lambda \sim 0.1$, radiates only by 10–20% weaker than it would in a continuous medium. Even in optics, not to mention longer waves, $r/\lambda \sim 0.1$ for a channel of radius $r \sim 5 \times 10^{-6}$ cm $\gg a \sim 3 \times 10^{-8}$ cm. The possibility to use a channel or a gap is of particular importance, not only for a moving charge, but also for an atom or another complex "system" (see section 6) which would be destroyed in a continuous medium. For suppressing ionization losses the trajectory of a charge or another radiator can simply be placed, instead of into a channel or a gap, in a vacuum outside the medium, but sufficiently close to it (in this case the Vavilov–Cherenkov radiation is small unless $d/\lambda \ll 1$, where d is the distance from the trajectory to the boundary between the medium and the vacuum).

The corresponding electrodynamic problems require rather cubersome calculations [18–20]. For methodical considerations it is instructive to note that in some cases it is useful to use the reciprocity theorem (see, for example, [10, section 89]). The application of this theorem makes it possible to establish that sufficiently thin channels or gaps do not influence the Vavilov-Cherenkov radiation caused by a charge [21] (see also [8, ch. 7]). By means of the same reciprocity theorem one finds that for dipoles and other multipoles even thin channels or gaps do, in general, affect the Vavilov–Cherenkov radiation. This problem is discussed in section 5.

The reciprocity theorem is also useful in solving other problems concerning radiators in a medium. For instance, let an oscillator (a dipole of frequency ω_0) be placed in the centre of an empty spherical cavity of radius $r \ll \lambda_0 = 2\pi c/\omega_0$ in a medium with permittivity $\epsilon(\omega)$. Then, on the basis of the known solution of the electrostatic problem concerning the field in a spherical cavity, the reciprocity theorem immediately suggests that the oscillator radiation field for the oscillator in a cavity differs by a factor of $\frac{3}{2}\epsilon(\omega_0)/(\epsilon(\omega_0)+1)$ from the radiation field in the case of continuous medium.

5. *Vavilov–Cherenkov radiation for electric, magnetic and toroidal dipoles*

Condition (4) for the opening of the cone for Vavilov–Cherenkov radiation has a kinematical or interferential character. The opening of the cone is therefore the same for all radiators—charges, dipoles, etc. (for anisotropic medium, see [22,23]). The intensity of radiation, its distribution and polarization along the cone depend on the character of the radiator.

Here we shall consider the Vavilov–Cherenkov radiation for the case of different dipoles. Although this problem has been discussed already for a long time [16,21,24,25], unclear points arose from time to time, which only recently have become clarified [26,27].

Let us write the field equations by means of which one can calculate the

Vavilov–Cherenkov radiation using the macroscopical method:

$$\operatorname{curl} \boldsymbol{H} = \frac{1}{c}\frac{\partial \epsilon \boldsymbol{E}}{\partial t} + \frac{4\pi}{c}\boldsymbol{j}, \tag{16}$$

$$\operatorname{curl} \boldsymbol{E} = -\frac{1}{c}\frac{\partial \mu \boldsymbol{H}}{\partial t}, \tag{17}$$

$$\operatorname{div} \epsilon \boldsymbol{E} = 4\pi\rho, \qquad \operatorname{div} \mu \boldsymbol{H} = 0. \tag{18}$$

We deal here with an everywhere non-moving medium whose properties are described by the permittivity ϵ and the permeability μ (ϵ and μ in eqs. (16)–(18) must be considered as operators, but for the problems considered one may at first put $\epsilon = \text{const}$, $\mu = \text{const}$, and in the final result substitute $\epsilon(\omega)$ and $\mu(\omega)$; see [8]). Besides, in the case of the Vavilov–Cherenkov effect the medium is assumed to be homogeneous.

If a source moves uniformly at velocity $\boldsymbol{v}$ and has a point charge e, an electric moment $\boldsymbol{p}$ and a magnetic moment $\boldsymbol{m}$, then

$$\boldsymbol{j} = e\boldsymbol{v}\delta(\boldsymbol{r} - \boldsymbol{v}t) + \frac{\partial}{\partial t}\{\boldsymbol{p}\delta(\boldsymbol{r} - \boldsymbol{v}t)\} + c \operatorname{curl}\{\boldsymbol{m}\delta(\boldsymbol{r} - \boldsymbol{v}t)\} \tag{19}$$

Here, $\boldsymbol{p}$ and $\boldsymbol{m}$ are moments in the laboratory frame of reference (by definition it coincides with a non-moving medium). In the rest frame of the source the values $\boldsymbol{p}'$ and $\boldsymbol{m}'$ are different (see below). If there exists only a charge, the solution of eqs. (16) and (17) leads to eq. (6). We will not write here all the equations for the case of dipoles (see refs [25,26]), but we will give those which are connected with unclear points. For a magnetic dipole $\boldsymbol{m}_\perp$ perpendicular to $\boldsymbol{v}$, we have

$$\left(\frac{\mathrm{d}W}{\mathrm{d}t}\right)_{m_\perp} = \frac{m_\perp^2}{2vc^2}\int \epsilon\mu^2 \left[2\left(1 - \frac{1}{\epsilon\mu}\right)^2 - \left(1 - \frac{v^2}{\epsilon\mu c^2}\right)\left(1 - \frac{c^2}{\epsilon\mu v^2}\right)\right]\omega^3\,\mathrm{d}\omega. \tag{20}$$

The dipole considered here is purely magnetic in the rest frame: in the laboratory frame it possesses also an electric dipole moment $\boldsymbol{p}_\perp = (1/c)[\boldsymbol{v}\boldsymbol{m}_\perp]$ (for clarity we should recall that in this case $\boldsymbol{m}' = \boldsymbol{m}$, $\boldsymbol{p}' = 0$). For an electric dipole $\boldsymbol{p}_\perp$ perpendicular to $\boldsymbol{v}$ (in this case $\boldsymbol{m}' = 0$, $\boldsymbol{m}_\perp = -(1/c)[\boldsymbol{v}\boldsymbol{p}_\perp]$) we have

$$\left(\frac{\mathrm{d}W}{\mathrm{d}t}\right)_{p_\perp} = \frac{p_\perp^2 v}{2c^4}\int \epsilon\mu^2\left(1 - \frac{c^2}{\epsilon\mu v^2}\right)^2 \omega^3\,\mathrm{d}\omega. \tag{21}$$

The magnetic dipole considered above [see eqs. (19) and (20)] is a usual "current" magnetic dipole. But in principle, there may exist also magnetic dipoles of other types (let us call them "true" magnetic dipoles) which are formed by two magnetic monopoles $+g$ and $-g$ [the moment of such a dipole is $\tilde{\boldsymbol{m}} = g\boldsymbol{d}$, where $\boldsymbol{d} = \boldsymbol{r}_2 - \boldsymbol{r}_1$, $\boldsymbol{r}_2$ and $\boldsymbol{r}_1$ being positions of the monopoles $+g$ and g (see fig. 2); of course, for a point dipole $d \to 0$, $gd = \tilde{m}$].

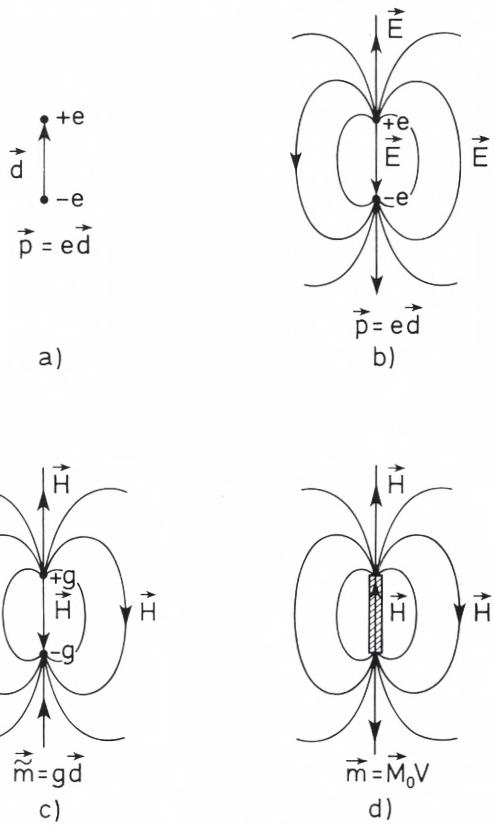

Fig. 2. (a, b) Electric dipole $\boldsymbol{p}$, (c) "true" magnetic dipole $\tilde{\boldsymbol{m}}$ and (d) "current" magnetic dipole $\boldsymbol{m}$. The current magnetic dipole is schematically shown here as a rod with magnetization $\boldsymbol{M}_0$ in a volume V; another possible model is a ring with a current.

To calculate the fields of magnetic monopoles *, dipoles, etc. in a medium, we can in a certain approximation, which we just use here, assume $\boldsymbol{j}=0$, $\rho=0$ in eqs. (16)–(18), but add to the right-hand side of eq. (17) the term $-(4\pi/c)\boldsymbol{j}_{\mathrm{m}}$, where $\boldsymbol{j}_{\mathrm{m}}$ is the current density of the magnetic monopoles (besides, the second of eqs. (18) takes the form div $\mu\boldsymbol{H}=4\pi\rho_{\mathrm{m}}$, where ρ_{m} is the magnetic charge density).

If the solution of the problem for an electric charge is known, the solution for a magnetic monopole is obtained from the duality principle (for more details see [26]). Thus, for the Vavilov–Cherenkov radiation of a magnetic monopole, where $\boldsymbol{j}_{\mathrm{m}}= g\boldsymbol{v}\delta(\boldsymbol{r}-\boldsymbol{v}t)$ we obtain from eq. (6):

$$\left(\frac{\mathrm{d}W}{\mathrm{d}t}\right)_g=\frac{g^2v}{c^2}\int\epsilon\left(1-\frac{c^2}{\epsilon\mu v^2}\right)\omega\,\mathrm{d}\omega. \tag{22a}$$

* For magnetic monopoles the electrodynamics of continuous media is, in general, more complicated than indicated here [39], but this is not especially important for us.

In the case of a "true" magnetic dipole $\tilde{\boldsymbol{m}}_{\perp}$, which is a direct analogue of an electric dipole $\boldsymbol{p}_{\perp}$, we similarly obtain from eq. (21):

$$\left(\frac{\mathrm{d}W}{\mathrm{d}t}\right)_{\tilde{m}_{\perp}} = \frac{\tilde{m}_{\perp}^2 v}{2c^4} \int \mu\epsilon^2 \left(1 - \frac{c^2}{\epsilon\mu v^2}\right)^2 \omega^3 \, \mathrm{d}\omega. \tag{22b}$$

Expression (22b) differs from eq. (20) which refers to the "current" dipole $\boldsymbol{m}_{\perp}$. This result [25] seems paradoxical, because the fields of a current and a true magnetic dipole are quite similar, at least outside the dipoles. But in fact these fields are not quite equivalent. For instance the field of a resting true dipole $\tilde{\boldsymbol{m}}$ is as follows:

$$\boldsymbol{H} = \frac{3(\tilde{\boldsymbol{m}}\boldsymbol{r})\boldsymbol{r} - r^2\tilde{\boldsymbol{m}}}{\mu r^5}, \quad \boldsymbol{E} = 0, \tag{23}$$

which is analogous to the field of an electric dipole

$$\boldsymbol{E} = \frac{3(\boldsymbol{p}\boldsymbol{r})\boldsymbol{r} - r^2\boldsymbol{p}}{\epsilon r^5}, \quad \boldsymbol{H} = 0. \tag{24}$$

At the same time the field of a current dipole is

$$\boldsymbol{H} = \frac{3(\boldsymbol{m}\boldsymbol{r})\boldsymbol{r} - r^2\boldsymbol{m}}{r^5} + 4\pi\boldsymbol{m}\delta(\boldsymbol{r}), \quad \boldsymbol{E} = 0. \tag{25}$$

The absence of the factor $1/\mu$ in the first term of this expression as compared with eq. (23) is associated with different definitions of the moments $\boldsymbol{m}$ and $\tilde{\boldsymbol{m}}$ [26]. The term $4\pi\boldsymbol{m}\delta(\boldsymbol{r})$ in eq. (25) reflects the difference between the dipoles, which is clear from fig. 2 (for a current dipole the field lines are closed; eq. (25) is the solution of eq. (16) with $\boldsymbol{j} = c \operatorname{curl}\{\boldsymbol{m}\delta(\boldsymbol{r})\}$).

If monopoles move in a medium with rather thin empty channels and gaps, the Vavilov–Cherenkov radiation remains unchanged (see section 4). In the case of dipoles, however, an arbitrarily thin channel or a gap, generally speaking, changes the field and the energy radiated by the source.

The presence of thin channels and gaps does not affect dipoles parallel to the velocity $\boldsymbol{v}$ (and, thus, to the channel axis or to the gap plane). This can be seen by means of the reciprocity theorem [21].

An electric dipole $\boldsymbol{p}_{\perp}$ perpendicular to $\boldsymbol{v}$ radiates in a thin cylindrical channel $[2\epsilon/(\epsilon+1)]^2$ times more than in a continuous medium; for a gap the amplification is characterized by a factor ϵ^2 (above we have put $\mu = 1$ and assumed that in the laboratory frame of reference the magnetic moment $\boldsymbol{m} = 0$; see [21] and [8, ch. 7]). This fact shows that a narrow surrounding is important for radiation by a dipole. Moreover, it was shown in [28] that a "current" dipole moment radiates like a "true" one, if the medium inside it is not at rest but is moving at the same velocity $\boldsymbol{v}$ as the dipole itself. I should confess, nonetheless, that I have not understood the

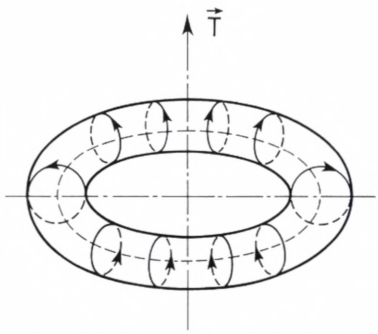

Fig. 3. Toroid with current. $\boldsymbol{T}$ is the toroidal dipole moment.

situation quite well until recently. The consideration of the Vavilov–Cherenkov radiation for a toroidal dipole moment [26,27] has provided a good insight into this matter. An example of a toroidal dipole is a "toroid", i.e. a toroid-like solenoid with current (fig. 3). If such a system is not charged ($\rho = 0$), it does not possess an electric dipole and higher multipole moments. Further, if an azimuthal current is absent (to this end the winding must be, for example, double—it must first go in one and then in the other direction), the system does not possess a magnetic moment either. But inside the toroid the field $\boldsymbol{H} \neq 0$ and the system has a toroidal dipole moment. Its general definition is as follows:

$$\boldsymbol{T} = \frac{1}{10c} \int \{(\boldsymbol{rj})\boldsymbol{r} - 2r^2\boldsymbol{j}\} \, \mathrm{d}^3\tau, \tag{26}$$

where $\boldsymbol{j}$ is the current density.
If a point toroidal dipole is at rest, the density of the toroidal moment $\mathscr{T} = \boldsymbol{T}\delta(\boldsymbol{r})$ and

$$\boldsymbol{j} = c \, \mathrm{curl} \, \mathrm{curl}\{\boldsymbol{T}\delta(\boldsymbol{r})\}. \tag{27}$$

If a toroidal dipole is moving in a vacuum at constant velocity $\boldsymbol{v}$, then, as follows from Lorentz transformations, outside the dipole the fields $\boldsymbol{H}$ and $\boldsymbol{E}$ are as before equal to zero. Inside the dipole the field $\boldsymbol{H} \neq 0$ and the field $\boldsymbol{E} = -(1/c)[\boldsymbol{vH}]$.

The fields of a toroidal dipole moving uniformly in a medium cannot, of course, be found by means of Lorentz transformations. The calculation on the basis of eqs. (16) and (17) with the current eq. (27) shows that outside such a toroidal dipole fields are present [27], and if condition (5) holds, the Vavilov–Cherenkov radiation appears with a power

$$\left(\frac{\mathrm{d}W}{\mathrm{d}t}\right)_{T_\parallel} = \frac{T^2}{c^4 v} \int \mu(\epsilon\mu - 1)^2 \left(1 - \frac{c^2}{\epsilon\mu v^2}\right) \omega^5 \, \mathrm{d}\omega. \tag{28}$$

The dipole $\boldsymbol{T}$ is here assumed to be directed along the velocity $\boldsymbol{v}$ and, what is essential, in the rest frame there exists only a toroidal dipole [i.e. the current density in the rest frame is given by eq. (27)]. If in the laboratory frame of reference

$$\boldsymbol{j} = c \text{ curl curl}\{\boldsymbol{T}_v \delta(\boldsymbol{r} - \boldsymbol{v}t)\},$$

then in the rest frame there also exists a quadrupole electric moment, and we obtain for $(\mathrm{d}W/\mathrm{d}t)_{T_{v,\parallel}}$ an expression [26,27] which differs from eq. (28) by the replacement of $(\epsilon\mu - 1)^2$ by $(\epsilon\mu)^2$ and, of course, by the replacement of T by T_v. By virtue of what has been said one should expect that if a toroidal dipole moves in an empty channel, the Vavilov–Cherenkov radiation completely vanishes. The calculation, naturally, confirms this conclusion [20,30]. For an empty channel or a gap the calculation is, however, not needed—the result is obvious because the field in vacuum, outside a toroidal dipole is equal to zero. But the calculation is necessary if the channel is filled with a medium with permittivity ϵ_0 and permeability μ_0. Then for a thin channel we obtain expression (28) with the replacement of $(\epsilon\mu - 1)^2$ by $(\epsilon_0\mu_0 - 1)^2$, which vanishes as $\epsilon_0\mu_0 \to 1$ and, of course, as $\epsilon_0 \to 1$, and $\mu_0 \to 1$.

What does all this mean? The whole point is that in the case considered the medium fills up the toroidal dipole inside which there is a field. This field affects, that is, polarizes the medium, and this effect lasts also after the dipole has passed. In other words, the dipole leaves a "trace". This picture is especially good for a plasma. Particles (electrons, ions) passing through a dipole are deflected inside it by the field, and therefore the plasma behind the dipole is perturbed. For a toroidal dipole this effect manifests itself, so to say, in a pure form. This also occurs with magnetic moments, and, due to different fields inside a "current" dipole and a "true" dipole [see eqs. (23, (25)] the corresponding "traces" in the medium are also different. According to this conclusion, a "current" dipole and a "true" magnetic dipole moving in an empty channel or in a gap must radiate similarly [21].

How will different dipoles radiate when they move in a continuous medium? Since the radiation of dipoles is already influenced by the motion in thin channels, and while the fields inside the dipoles are essential, it is clear that not only "far" collisions, but also "near" collisions are responsible for the radiation. If the dipoles are macroscopic, then the passage of a medium (for example, plasma) through the dipoles may play some role. But for microscopic (point) dipoles the role of "near" collisions may be considered only in the framework of the microtheory or with an account of spatial dispersion. This also concerns the problems dealing with particle radiation in the channelling process. If we omit channelling of charged particles, the corresponding problems are of no real importance (at least at the present time) simply because of the smallness of the dipole moments of different particles (neutrons, etc.) Radiation by toroidal dipoles moving in a medium may apparently be only of methodical interest. In solid-state theory crystals with a toroidal moment of nonzero density are, however, specific magnetic substances [32].

6. *The classical and quantum theories of the Doppler effect in a medium*

The quantum theory of the Vavilov–Cherenkov effect in the classical limit (14) does not give anything new, apart from an understanding of the role played by the conservation laws. It is therefore interesting that in more complicated cases quantum theory enables us to reveal interesting points even in the classical limit. To illustrate this, we consider the Doppler effect in a medium.

First, we recall the classical situation using as an example an oscillator with frequency ω_{00} (this is a frequency in a frame of reference in which the oscillator as a whole is at rest). If the oscillator moves in vacuo with a constant velocity $\boldsymbol{v}$ (in the laboratory frame of reference), then in this frame of reference the frequency of the waves emitted by it equals

$$\omega(\theta)=\frac{\omega_{00}\sqrt{1-v^2/c^2}}{1-(v/c)\cos\theta}=\frac{\omega_0}{1-(v/c)\cos\theta}, \tag{29}$$

where θ is the angle between the wavevector (in the direction of observation) and $\boldsymbol{v}$, and ω_0 is the oscillator frequency in the laboratory frame.

Now let there be a transparent medium [with a refractive index $n(\omega)$], which is at rest in the laboratory frame of reference. We should not be disturbed by the fact that the motion of the source in a medium may lead to large energy losses and, which is important, to the destruction of the source itself (say, of an excited atom). Indeed, as pointed out in section 4, to eliminate losses and destructive collisions, one can make an empty gap or an empty channel in the medium or direct a beam of atoms near the boundary between the medium and the vacuum.

If a medium is present, eq. (29) is replaced by

$$\omega(\theta)=\frac{\omega_{00}\sqrt{1-v^2/c^2}}{\left|1-\frac{v}{c}n(\omega)\cos\theta\right|}=\frac{\omega_0}{\left|1-\frac{v}{c}n(\omega)\cos\theta\right|}. \tag{30}$$

One can obtain eq. (30) from eq. (29) by using the general rule consisting in the replacement of v/c by vn/c (in the expression $\sqrt{1-v^2/c^2}$ one should not, of course, make this substitution, as it does not concern the emission process). However, one can obtain eq. (30) also by solving the problem of an oscillator moving in a medium. It is nontrivial that absolute values occur in eq. (30). If the motion is subluminal $(v<c/n)$ or if in the case of a superluminal motion the emission proceeds outside the cone [eq. (4)], i.e.

$$\frac{v}{c}n(\omega)\cos\theta<1, \tag{31}$$

we are dealing with the usual, normal Doppler effect. The so-called complex Doppler effect due to dispersion (i.e. due to the dependence of n on ω) is also possible in this case (the complex Doppler effect as well as eq. (30) with the modulus were first considered by Frank in 1942 [24]).

If the motion is superluminal, then under the condition

$$\frac{v}{c}n(\omega)\cos\theta>1 \tag{32}$$

[i.e. when there is emission into the cone (eq. 4) which is often referred to as the

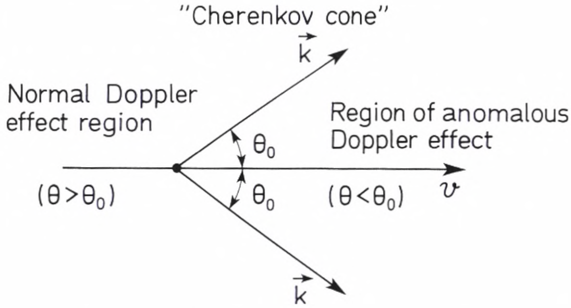

Fig. 4. Regions of normal and anomalous Doppler effect.

"Cherenkov cone"; see fig. 4] eq. (30) without the modulus would give negative values of the frequency ω. From this it is clear that it is necessary to introduce the absolute signs.

Under condition (32) the Doppler effect is called anomalous. If dispersion is taken into account, the whole picture is rather complicated. Since here we are interested in another aspect of the question, we shall neglect dispersion. In this case it follows from eq. (30) with $n(\omega)=n=$ const. that, on the Cherenkov cone itself [for $(vn/c)\cos\theta=(vn/c)\cos\theta_0=1$; see eq. (4)] the frequency $\omega(\theta_0)=\infty$ and $\omega(\theta)\to\infty$ as $\theta\to\theta_0$ on both sides of the cone. It is impossible to say anything more on the basis of eq. (30), and the difference between the normal and the anomalous Doppler effects does not seem to be a profound one.

We now turn to a quantum derivation of the formula for the Doppler effect in a medium [33]. To do this, one should use the conservation laws (9) and (11), but replacing the particle energies $E_{0,1}$ for a charge (or, more generally, for a source without internal degrees of freedom) by the expression $E_{0,1}=\sqrt{(m+m_{0,1})^2c^4+c^2p_{0,1}^2}$, where $(m+m_0)c^2=mc^2+W_0$ is the total energy of the system (atom) in the lower state, 0, and $(m+m_1)c^2=mc^2+W_1$ is the total energy in the upper state, 1.

For simplicity we consider here a two-level system and refer to the state with a larger energy as the upper state [that is, $W_1>W_0$, and the frequency of the radiation by an atom at rest is $\omega_{00}=(W_1-W_0)/\hbar$].

If we now apply the conservation laws in the classical limit (14) and the relation $\Delta E=E_1-E_0=\boldsymbol{v}\Delta\boldsymbol{p}$, we arrive at eq. (30). A more general calculation, which takes into account recoil, is presented in ref. [33]. It is, however, not on account of the quantum corrections that this is of importance, but because of the fact that, tracing down the signs one can observe an important point which is, of course, completely hidden in the classical derivation of eq. (30): In the region of the normal Doppler effect [eq. (31)] the emission at frequency ω corresponds to a transition of the atom from the upper state 1 to the lower state 0 (the direction of the transition is determined from the requirement that the energy $\hbar\omega$ of the emitted quantum should be positive, that is from the requirement $\omega>0$). In the case of an anomalous Doppler effect [eq. (32)], i.e. if a quantum of energy $\hbar\omega$ is emitted into the Cherenkov cone, the atom must transit from below (state 0) upwards (to state 1)

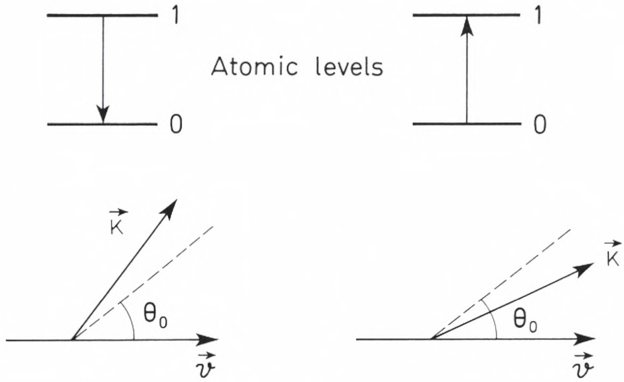

Fig. 5. Transitions between levels 0 and 1 in the case of a normal and an anomalous Doppler effect.

(For some more details see also fig. 5 and refs. [33,34].) There is no contradiction here—the energy that goes to the excitation of the radiating system (the atom), as well as the radiation energy $\hbar\omega$ itself, is derived from the kinetic energy of translational motion.

Therefore, in the case of superluminal motion $v > c/n$, where the anomalous Doppler effect is possible, the radiating atom, even if initially not excited will get excited with the simultaneous emission of quanta inside the Cherenkov cone. Making a transition to a lower state, the excited atom emits quanta at angles $\theta > \theta_0$, i.e. outside the Cherenkov cone. In this connection see also [40].

I think it would have been rather difficult to establish this unusual picture without quantum calculation. We can confirm the result and develop the theory further using the classical calculations of the radiative friction force acting upon a superluminal oscillator. In accordance with the above conclusion, the waves emitted outside the Cherenkov cone lead to a damping of the oscillator vibrations, whereas the waves emitted inside the cone (the anomalous Doppler effect) pump up the oscillator vibrations (see ref. [8, ch. 7] and the literature cited therein).

7. Transition radiation at the boundary between two media

If the medium is inhomogeneous and (or) changes in time or if such a medium lies near the trajectory of a source, transition radiation occurs. This radiation originates when a charge (or another source with zero eigenfrequency) moves uniformly and rectilinearly under non-uniform conditions—in an inhomogeneous or a time-dependent medium (or near such media). Transition radiation in general may coexist and interfere with the Cherenkov radiation and with the radiation due to charge acceleration (i.e. with bremsstrahlung, synchrotron radiation, etc.). But for a deeper insight into the physics of the case, we consider the transition radiation alone.

Let a charge move at a constant velocity

$$v < \frac{c}{n}, \tag{33}$$

such that Vavilov–Cherenkov radiation does not occur. If, besides, we are dealing with a vacuum ($n=1$), there is no radiation at all. For the radiation to appear in a vacuum, a charge (or a multipole) must be accelerated or, in other words, the parameter v/c, which characterizes the radiation, must change. If the medium is transparent, this parameter has the form $v/c_{\text{ph}} = vn(\omega)/c$. This is equal to the ratio of the particle velocity v to the phase velocity of light, $c_{\text{ph}} = c/n(\omega)$.

The radiation occurring when the parameter vn/c changes due to changes in n along or close to the trajectory of a source with $\boldsymbol{v} = \text{const}$ is called transition radiation. To be precise, in the general case of an absorbing medium the role of the refractive index n is played by $\sqrt{\epsilon} = n + \mathrm{i}\kappa$, where ϵ is the complex dielectric permittivity of the medium (for simplicity we assume here, and below, that the medium is nonmagnetic, i.e. $\mu = 1$).

The simplest problem of this kind is a charge crossing the boundary between two media considered by I.M. Frank and the present author in 1944 [35]. Transition radiation is an even simpler effect than Vavilov–Cherenkov radiation. The reason for the possibility of transition radiation to be revealed so late is the same as in the case of the Vavilov–Cherenkov effect.

It is useful to recall the most obvious explanation of the reason for the occurrence of transition radiation when a charge crosses the boundary between media. It is well known that the electromagnetic field in the first medium (in the medium in which the charge moves at a given moment of time) can be represented as the field of the charge itself and the field of its "mirror image" moving in the second medium towards the charge. When the charge and its image cross the boundary, they partially "annihilate" as it were "from the point of view" of the first medium, and "reconstruct themselves"; and this leads to the radiation. Especially simple is the case of a charge impinging normally on an ideal mirror. When crossing the boundary of the mirror, the charge and its image $-e$ "annihilate" each other completely or, rather, stop at the boundary (in the sense that the radiation occurring in a vacuum is the same as the radiation by an incident charge e and its image $-e$ which simultaneously stop at the boundary; c.f. fig. 6).

To find the emitted energy W in this simplest case, we need not solve the rather cumbersome boundary problem, but we can use a simple formula for radiation by charges which suddenly change their velocity:

$$W(\omega, \theta, \varphi) = \frac{1}{4\pi^2 c^3}\left\{\sum_i e_i\left(\frac{[\boldsymbol{v}_{i2}\boldsymbol{s}]}{1-(\boldsymbol{v}_{i2}\boldsymbol{s})/c} - \frac{[\boldsymbol{v}_{i1}\boldsymbol{s}]}{1-(\boldsymbol{v}_{i1}\boldsymbol{s})/c}\right)\right\}^2. \tag{34}$$

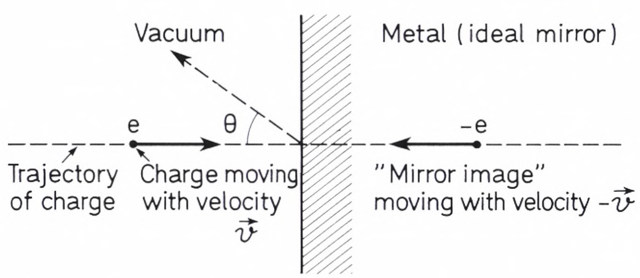

Fig. 6. Transition radiation of a charge e crossing the boundary between vacuum and a metal.

Here e_i is the charge of the ith particle, whose velocity changes sharply from $\boldsymbol{v}_{i1}$ to $\boldsymbol{v}_{i2}$, and $\boldsymbol{s} = \boldsymbol{k}/k$ is the direction of the radiation wavevector characterized by the angles θ and φ; the total frequency-dependent energy density of the radiation is $W(\omega) = \int W(\omega, \theta, \varphi) \sin\theta \, d\theta \, d\varphi$, and the total energy $W = \int W(\omega) \, d\omega$.

If in a vacuum (in the absence of boundaries, etc.) one charge $e_1 = e$ stops abruptly or accelerates rapidly from the rest state to velocity v, then [see eq. (34)],

$$W(\omega, \theta, \varphi) = \frac{e^2 v^2 \sin^2\theta}{4\pi^2 c^3 [1 - (v/c)\cos\theta]^2},$$

$$W(\omega) = \frac{e^2}{\pi c}\left(\frac{1}{v/c}\ln\frac{1 + v/c}{1 - v/c} - 2\right). \tag{35}$$

In the case of transition radiation on an ideal mirror, one assumes in eq. (34) that the charge $e_1 = e$ with velocity $\boldsymbol{v}$ and the charge $e_2 = -e$ with velocity $-\boldsymbol{v}$ stop abruptly at the boundary (see fig. 6). As a result, one observes radiation in medium 1 (in vacuo) with energy

$$W_1(\omega, \theta) = \frac{e^2 v^2 \sin^2\theta}{\pi^2 c^3 \left[1 - (v/c)^2 \cos^2\theta\right]^2},$$

$$W_1(\omega) = \frac{e^2}{\pi c}\left[\frac{1 + (v/c)^2}{2v/c}\ln\frac{1 + v/c}{1 - v/c} - 1\right]. \tag{36}$$

Here θ is the angle between $\boldsymbol{k}$ and $-\boldsymbol{v}$, as shown in fig. 6. In the nonrelativistic case (i.e. for $v \ll c$)

$$W_1(\omega, \theta) = \frac{e^2 v^2 \sin^2\theta}{\pi^2 c^3}, \qquad W_1(\omega) = \frac{4e^2 v^2}{3\pi c^3}. \tag{37}$$

In the ultrarelativistic case ($v \to c$)

$$W_1(\omega) = \frac{e^2}{\pi c}\ln\frac{2}{1 - v/c} = 2\frac{e^2}{\pi c}\ln\frac{2E}{mc^2},$$

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \gg mc^2, \tag{38}$$

which coincides with the radiation of a single particle in the same limit [see eq. (35)]. The latter is clear since, when $v \to c$, the radiation is directed along the charge velocity, and therefore the radiation of the charge e "going into metal" is not observed (in the medium 1—in vacuo—one observes only the radiation of the mirror image, of the charge $-$e, with a velocity $-\boldsymbol{v}$). In the nonrelativistic limit the energy (37) is four times larger than the energy (35) for a single charge radiating into the backward hemisphere (into the vacuum), because for a nonrelativistic velocity the fields of the charge e and of its mirror image $-e$ add up, i.e. they are doubled.

The transition radiation discussed here occurs if a boundary between two media with different "electrical" parameters (e.g. dielectric permittivity, refractive index) is crossed. However, initially attention was concentrated on the incidence of a charge on a metal (which may not be a perfect mirror) and, hence, on transition radiation —mainly optical—in the backward direction, which is observed in vacuum. For relativistic particles of sufficiently high energy it is, however, quite realistic that a particle may well pass through a medium and go into a vacuum. This problem is equivalent to the previous one, and the corresponding formula for the radiation intensity is simply derived by a replacement of the velocity v by $-v$ (see below). At the same time, in the calculation of fields there is no symmetry in these cases and the radiation intensities are different when we replace v by $-v$, and under certain conditions the differences are large. For forward radiation and, in particular, when a particle leaves the medium and enters the vacuum, the radiation spectrum has higher frequencies. In a condensed medium the transition radiation of relativistic particles can stretch out into the X-ray part of the spectrum. It is not expedient to give here the solution of the corresponding boundary problems (see refs. [10–12] and the literature cited therein).

The final formula for the case where medium 1 is a vacuum and medium 2 is described by the complex permittivity ϵ [35], is given by

$$W_1(\omega, \theta) = \frac{e^2v^2 \sin^2\theta \cos^2\theta \left|(\epsilon - 1)\left[1 - v^2/c^2 + (v/c)\sqrt{\epsilon - \sin^2\theta}\right]\right|^2}{\pi^2c^3\left|\left[1 - (v^2/c^2)\cos^2\theta\right]\left[1 + (v/c)\sqrt{\epsilon - \sin^2\theta}\right]\left(\epsilon \cos\theta + \sqrt{\epsilon - \sin^2\theta}\right)\right|^2}. \tag{39}$$

For an ideal mirror one can assume that $|\epsilon| \to \infty$ and eq. (39) goes over into eq. (36), as should be the case. The expression for $W_2(\omega, \theta)$, referring to the case where the charge e leaves a medium of permittivity ϵ (medium 1) for a vacuum (medium 2) with velocity v, is derived from eq. (39) by a replacement of v by $-v$; besides, the angle θ is now the angle between $\boldsymbol{k}$ and $\boldsymbol{v}$ [but not beteen $\boldsymbol{k}$ and $-\boldsymbol{v}$, as in eq. (39)]. The replacement of v by $-v$ in eq. (39) is not at all innocent—in the denominator there appears a factor $[1 - (v/c)\sqrt{\epsilon - \sin^2\theta}\,]$, instead of $[1 + (v/c)\sqrt{\epsilon - \sin^2\theta}\,]$. This is just the reason why there appear higher frequencies in the radiation spectrum when a particle leaves the medium (we must take into account that ϵ approaches unity for high frequencies). As a result the total intensity (integrated over all angles and frequencies) also increases; in the simplest case it turns out to be proportional to

$$\frac{E}{mc^2} = \frac{1}{\sqrt{1 - v^2/c^2}}$$

(see section 10 below; E is the total energy of a radiating charge of mass m). This important fact was clarified in 1959 by Barsukov and Garibyan [36]. This opened up much wider perspectives for the creation of efficient "transition counters" intended for determination of relativistic particle energies.

The appearance of a possibility for a practical application to high-energy physics, stimulated interest to transition radiation. Altogether, transition radiation, this rather simple and clear effect in the field of classical electrodynamics, had hardly attracted any attention for about 15 years; now it is very popular, although mainly in the context of developing and applying transition counters. The latter problem has been mentioned even in a number of review articles, apart from a large number of papers (for references see [12,38]).

8. *Transition radiation as a more general phenomenon; formation zone*

Transition radiation in the broad sense is also of value from a general physical point of view. It develops certain ideas and a "language", and thereby facilitates further developments. The situation here is similar to the one with the Vavilov–Cherenkov effect used in Cherenkov counters.

Our attention has so far been concentrated on the transition radiation which occurs when one or several boundaries between media are crossed. In the latter case we deal either with an ordered sequence of boundaries, i.e. a system with a definite period, or with randomly distributed boundaries (inhomogeneities). Another trend which has developed is based on the fact that any radiation is formed not in a point, but in some region (the formation zone) whose dimension is determined by the wavelength λ, but its dimension can also be appreciably larger. This is the reason why the Vavilov–Cherenkov effect occurs when a particle moves in a vacuum but near a medium (in a channel, a gap or near a boundary between media). Quite similarly, transition radiation (which in this case is called diffraction radiation) occurs when a source (charge) moving uniformly in a vacuum (or in a uniform medium) passes close to some obstacles—metallic or dielectric globules, diaphragms, a diffraction grating, etc. Apart from the above general remark, the occurrence of this transition radiation can also be explained on the basis of the method of images.

For relativistic particles, when we consider radiation in the direction of their velocity, the formation zone generally increases with an increasing particle energy. For instance, in vacuo the size of the formation zone L_{f} in the direction of the velocity for a given radiation wavelength λ increases proportionally to

$$\left(\frac{E}{mc^2}\right)^2 = \left(1 - \frac{v^2}{c^2}\right)^{-1},$$

where E is the total charge (source) energy and it is assumed that $E \gg mc^2$.

The concept of the radiation formation zone and its size L_{f}, and the concept of the radiation formation time $t_{\mathrm{f}} = L_{\mathrm{f}}/v$, are of great importance not only in electrodynamics, but also in high-energy physics (see [11,24] and [37, section 93]).
The derivation of the expressions for L_{f} and t_{f} for a source moving at velocity v in a transparent medium with a refractive index $n(\omega)$ and emitting waves at an angle θ

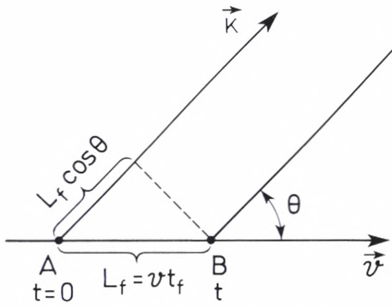

Fig. 7. The length L_f of the radiation formation zone.

to $\boldsymbol{v}$ (fig. 7) seems worth considering here. Let at the moment $t=0$ the source be at a point A and the phase of the wave emitted by it in direction $\boldsymbol{k}$ be equal to φ_A. We define the formation time t_f as the time after which the phase of the wave φ_B emitted at a point B in the same direction $\boldsymbol{k}$ differs from the phase of the wave φ_A emitted at point A by 2π. The phase factor of the wave has the form $\exp i\varphi = \exp i(\boldsymbol{kr} - \omega t)$. As is clear from fig. 7,

$$|\varphi_A - \varphi_B| = |kL_f \cos\theta - \omega t_f| = \left|\frac{\omega n}{c} v t_f \cos\theta - \omega t_f\right|, \tag{40}$$

since the size of the formation zone is the path $L_f = vt_f$. It follows from eq. (40) that

$$L_f = vt_f = \frac{2\pi v}{\omega|1-(v/c)\, n(\omega)\cos\theta|} = \frac{(v\, n(\omega)/c)\lambda}{|1-(v/c)\, n(\omega)\cos\theta|}. \tag{41}$$

The meaning of the formation time t_f is especially obvious when we deal with forward radiation in a vacuum: in this case, within the formation time t_f the radiation is ahead of the particle by a wavelength λ [indeed, in a vacuum for $\theta=0$, according to eq. (41),

$$t_f = \frac{\lambda}{c(1-v/c)} \quad \text{and} \quad (c-v)t_f = \lambda = \frac{2\pi c}{\omega};$$

when we pass over to a medium, we should replace c by $c_{ph} = c/n$ (since we deal with phase relations) and from

$$\left(\frac{c}{n} - v\right)t_f = \lambda = \frac{2\pi c}{\omega n}$$

we obtain expression (41) for $\theta = 0$]. In a vacuum, for $\theta = 0$, the formation length

$$L_f = \frac{\lambda \cdot v/c}{1-v/c},$$

and for $v \to c$, as has already been mentioned,

$$L_f = 2\lambda \left(\frac{E}{mc^2} \right)^2, \quad E \gg mc^2. \tag{42}$$

For sufficiently high energies the dimension of the formation zone L_f and time t_f can increase and finally exceed greatly the wavelength λ and time λ/v. For instance, the intensity of transition radiation from two boundaries, say, boundaries of a plate of thickness d may be considered (the interference terms being neglected) as a sum of the radiation intensities from one boundary only, provided that $L_f \ll d$. If $L_f \gtrsim d$ and especially if $L_f \gg d$, the radiation from a plate differs markedly from the radiation from two independent boundaries.

What has been said determines to a large extent the specificity of transition radiation by a pile of plates or, in general, by a regularly inhomogeneous medium (for details concerning this radiation, which was called the resonance transition radiation or transition scattering, see refs. [11,12]).

Another type of transition radiation occurs in a homogeneous but time-dependent medium. The essence of the matter is explained most easily in terms of the parameter vn/c. In order that a transition radiation can occur (for $\boldsymbol{v} = \text{const.}$), the refractive index must change on the charge trajectory or near to it. But this change will also take place if the index n changes in time. This kind of transition radiation can occur even for a charge which is at rest relative to the medium. Indeed, if by applying a magnetic or an electric field, or by some other means, one changes the medium rather sharply from an optically isotropic state to an anisotropic state, the polarization of the medium surrounding the fixed charge changes, thereby losing its spherical symmetry. This change in the polarization entails the emission of electromagnetic waves.

Like the Vavilov–Cherenkov radiation, the transition radiation is also of a very general character in the sense that it takes place for various kinds of waves. As an example we mention transition radiation of acoustic waves, arising when a moving dislocation crosses a grain boundary in a polycrystalline body. Other problems connected with transition radiation are also of interest in acoustics (see also section 11, below).

Interesting and may be of importance in applications to pulsar magnetospheres is transition radiation in the presence of a strong magnetic field leading to nonlinear electrodynamic effects.

9. *Transition scattering; transition bremsstrahlung*

If transition radiation occurs when a charge moves in a medium with a periodically (say, sinusoidally) changing refractive index, it may be called not only transition radiation or resonance transition radiation, but also transition scattering. A dielectric permittivity (refractive index) wave, which can be a standing or a travelling wave, is in this case scattered by a moving charge generating electromagnetic (transition) radiation. But the term transition scattering used in this case instead of transition radiation would be irrelevant, if the effect did not take place also in the limiting case of a charge at rest.

We consider an isotropic medium characterized by a dielectric permittivity ϵ which depends on the density ρ of the medium only. If a longitudinal acoustic wave propagates in this medium, the density is $\rho = \rho^{(0)} + \rho^{(1)} \sin(\boldsymbol{k}_0 r - \omega_0 t)$ and, by virtue of what has been said above,

$$\epsilon = \epsilon^{(0)} + \epsilon^{(1)} \sin(\boldsymbol{k}_0 \boldsymbol{r} - \omega_0 t), \tag{43}$$

where $\epsilon^{(1)}$ is the change in ϵ caused by the change in ρ (in the simplest case $\epsilon^{(1)} = \text{const.}\ \rho^{(1)}$). We have chosen a definite mechanism for the change in ϵ (in this case due to a change in the density ρ) to make our consideration concrete.

We now place in a medium a fixed (or, say, an infinitely heavy) charge e. Around this charge there appears an induction and a field

$$\boldsymbol{D}(\boldsymbol{r}, t) = \epsilon \boldsymbol{E}(\boldsymbol{r}, t), \quad \boldsymbol{D}^{(0)} = \frac{e\boldsymbol{r}}{r^3}, \quad \boldsymbol{E}^{(0)} = \frac{e\boldsymbol{r}}{\epsilon^{(0)} r^3}, \tag{44}$$

where the superscript (0) implies an "unperturbed" problem (the field of a charge without a permittivity wave). In the presence of a permittivity wave a varying polarization

$$\delta \boldsymbol{P} = \frac{\delta \boldsymbol{D}}{4\pi} = \frac{\epsilon^{(1)} \boldsymbol{E}^{(0)}}{4\pi} \sin(\boldsymbol{k}_0 \boldsymbol{r} - \omega_0 t) \tag{45}$$

arises around the charge, in a first approximation (corresponding to the assumption $|\epsilon^{(1)}| \ll \epsilon^{(0)}$). This polarization which possesses no spherical symmetry for $\boldsymbol{k}_0 \neq 0$, results in the appearance of an electromagnetic wave with a frequency ω_0, emanating from the charge (see fig. 8). The wave number of this wave is

$$k = \frac{2\pi}{\lambda} = \frac{\omega^{(0)}}{c} \sqrt{\epsilon^{(0)}} .$$

If, as we assumed, the permittivity wave is caused by the acoustic wave, $k \ll k_0 = \omega_0 / U$, where U is the sound velocity (we assume here that $U \ll c/\sqrt{\epsilon^{(0)}}$).

The electromagnetic wave may be regarded as being scattered in the same sense as for other kinds of scattering, such as, for instance, the scattering of an electromagnetic wave by an electron at rest (in this case, we mean a rest state, only when the effect of an incident wave is disregarded). Transition scattering plays a prominent role in plasma physics (see section 10, below) and is on the whole a rather general phenomenon which occurs in a vacuum when an electromagnetic or a gravitational wave is incident on the region with a strong constant (static) or quasi-stationary electromagnetic field.

Closely related to the transition scattering process is transition bremsstrahlung. It occurs in a medium if a uniformly and rectilinearly moving charge e passes close to another charge e at rest. Transition bremsstrahlung is similar in its characteristics to

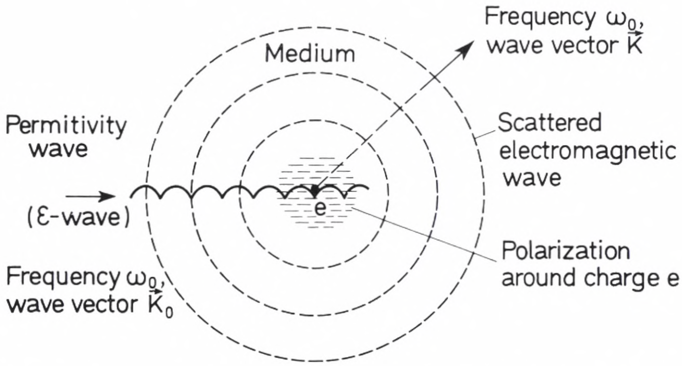

Fig. 8. Schematic picture characterizing the process of transition radiation formation on a non-moving (fixed) charge.

ordinary bremsstrahlung, although for its occurrence an acceleration (a change in the rectilinear trajectory or deceleration) is not necessary. The term "transition bremsstrahlung" is justified also because this radiation occurs in particle collisions. Moreover, transition bremsstrahlung of electrons is described by expressions which are very close to the corresponding formulae for ordinary bremsstrahlung. Further, transition bremsstrahlung interferes with ordinary bremsstrahlung. However, in contrast to ordinary bremsstrahlung, transition bremsstrahlung does not disappear in the limit of infinitely heavy colliding particles. The general theory of bremsstrahlung of particles in a medium should take into account also transition bremsstrahlung and its interference with ordinary bremsstrahlung (in the same way as the general theory of scattering must take into account transition scattering).

It is easy to understand the physical nature of transition bremsstrahlung if we bear in mind that the field $\boldsymbol{E}$ and the polarization $\boldsymbol{P} = (\epsilon - 1)\boldsymbol{E}/4\pi$ of a uniformly moving and, in particular, a resting charge can be expanded into waves with a wave vector $\boldsymbol{k}_0$ and a frequency $\omega_0 = (\boldsymbol{k}_0\boldsymbol{v})$, where $\boldsymbol{v}$ is the charge velocity. These waves are connected to permittivity waves with the same values $\boldsymbol{k}_0$ and ω_0. Such permittivity waves "dragged along" by a single charge may undergo transition scattering by another charge, as a result of which electromagnetic radiation, in this case transition bremsstrahlung, may be produced.

Transition bremsstrahlung may generate any "normal waves" (excitons, photons, phonons, and so on) which can propagate in the medium considered and is therefore a phenomenon of rather general character.

10. Transition radiation, transition scattering and transition bremsstrahlung in a plasma

Transition radiation, transition scattering and transition bremsstrahlung play a particularly important role for plasmas. In plasma physics one can go far by using a microscopical approach and avoid introducing the concept of transition scattering, but it may provide an insight into the situation and facilitate the development in this field [12].

In a rarified (in the collisionless limit) plasma the processes of transition scattering and transition bremsstrahlung may turn out to be particularly important. The above mentioned transition processes occur without any particle acceleration due to inhomogeneity of the medium along the trajectory of rectilinearly and uniformly moving charges. Collisions being neglected, the particle motion in a plasma is, in a first approximation, rectilinear and uniform if the action of macroscopic magnetic and electric fields are excluded. Various instabilities may appear in a collisionless plasma leading to a background of different kinds of waves. These waves are permittivity waves undergoing transition scattering. These permittivity waves are coupled with the most typical and most often observed high-frequency (Langmuir) wave in an isotropic plasma. Let us consider this example in more detail.

For a collisionless isotropic plasma the longitudinal dielectric permittivity has the form (see, for instance, [8, ch. 11]; the role of ions is assumed to be negligible small)

$$\epsilon_\ell(\omega, k) = 1 - \frac{\omega_p^2}{\omega^2} - 3\frac{\kappa T \omega_p^2 k^2}{m\omega^4},$$

$$\omega_p^2 = \frac{4\pi e^2 \mathcal{N}}{m}, \quad \omega^2 \gg \frac{\kappa T}{m}k^2, \tag{46}$$

where κ is Boltzmanns constant, T the temperature and $\mathcal{N}$ the electron density. The dispersion equation for longitudinal waves $\epsilon_\ell(\omega, k) = 0$ then leads to the following form for a longitudinal (plasma) wave

$$\boldsymbol{E} = \boldsymbol{E}_0 \cos(\boldsymbol{k}_0 \boldsymbol{r} - \omega_0 t), \quad [\boldsymbol{E}_0 \boldsymbol{k}] = 0, \quad \boldsymbol{E}_0 \boldsymbol{k} = E_0 k,$$

$$\omega_0^2 \approx \omega_p^2 + \frac{3\kappa T}{m}k^2. \tag{47}$$

Further,

$$\operatorname{div} \boldsymbol{E} = -E_0 k_0 \sin(\boldsymbol{k}_0 \boldsymbol{r} - \omega_0 t) = -4\pi e \mathcal{N}^{(1)}, \quad \mathcal{N} = \mathcal{N}^{(0)} + \mathcal{N}^{(1)}$$

(the electron charge is $-e$), from which it is clear that the longitudinal wave is coupled to the permittivity wave (the charge $-e\mathcal{N}^{(0)}$ is compensated by the ion charge)

$$\epsilon_\ell = \epsilon_\ell^{(1)} \sin(\boldsymbol{k}_0 \boldsymbol{r} - \omega_0 t),$$

$$\epsilon_\ell^{(1)} = -\frac{4\pi e^2 \mathcal{N}^{(1)}}{m\omega_0^2} = -\frac{e k_0 E_0}{m\omega_0^2} \tag{48}$$

Thus, plasma particles in a plasma wave are affected by the electric field $\boldsymbol{E}$ of the wave and at the same time a permittivity wave falls on them. The oscillations of the electrons in the field lead to Thomson scattering with the well-known cross-section

$$\sigma_T = \tfrac{8}{3}\pi r_e^2 = \tfrac{8}{3}\pi\left(\frac{e^2}{mc^2}\right)^2 = 6.65 \times 10^{-25}\ \text{cm}^2.$$

[we are dealing with a total cross-section for non-polarized transverse radiation; it is assumed that the electron velocity $v \ll c/n(\omega)$.] Simultaneously occurring transition scattering interferes with Thomson scattering. For electrons, both effects are of the same order of magnitude. In this respect a plasma is a complicated object since one should take into account both spatial and frequency dispersion. The corresponding formulae are presented in ref. [12]. In the case of ions the role of Thomson scattering is small due to the large ion mass, whereas transition scattering prevails and is of the same order of magnitude as for electrons. The same can be said about transition bremsstrahlung—it is as important for ions as it is for electrons, whereas ordinary bremsstrahlung is significant only for electrons.

At boundaries between media, transition radiation in a plasma is usually not so interesting because plasma boundaries are smeared out. This remark is, however, rather conditional since all depends on the wavelength of waves we are dealing with. For sufficient long waves, even if there are no walls (to say nothing of the conditions where such walls do exist) the plasma boundary may turn out to be sharp enough for the appearance of a noticeable transition effect.

For sufficiently large frequencies $\omega^2 \gg \omega_s^2$ (where ω_s is the eigenfrequency of the medium) all media obey in a good approximation, the plasma formula

$$\epsilon(\omega) = 1 - \frac{\omega_{p,t}^2}{\omega^2}, \quad \omega_{p,t}^2 = \frac{4\pi e^2 \mathcal{N}_t}{m}, \tag{49}$$

where $\mathcal{N}_t$ is the total electron concentration in the medium.

In the forward radiation of a relativistic particle leaving a plate, the energy is concentrated mainly in the X-ray range and, therefore, one can use eq. (49) to calculate the total energy W_2 for any medium. The result mentioned in section 6 is (see ref. [12])

$$W_2 = \frac{1}{3}\frac{e^2}{c}\omega_{p,t}\cdot\frac{E}{mc^2}, \tag{50}$$

the maximum of radiation corresponding to the frequency

$$\omega_m \sim \omega_{p,t}\cdot\frac{E}{mc^2}. \tag{51}$$

According to eq. (42) with $\lambda \sim \lambda_m$, the dimension of the formation zone is in this case,

$$\lambda_m \sim \frac{2\pi c}{\omega_m} \sim \frac{2\pi c}{\omega_{p,t}}\left(\frac{mc^2}{E}\right),$$

equal to

$$L_f \sim \frac{4\pi c}{\omega_{p,t}}\left(\frac{E}{mc^2}\right) \sim 10^{-5}\left(\frac{E}{mc^2}\right) \text{ cm}, \tag{52}$$

where we have taken into account that for an ordinary medium $\omega_{p,t} \sim 10^{16}$–$10^{17}$ $\sec^{-1}$ and $\lambda_{p,t} = 2\pi c/\omega_{p,t} \sim 10^{-5}$–$10^{-6}$ cm. For high energy particles, for example, for protons with $E \sim 10^{15}$ eV, $E/mc^2 \sim 10^6$ and $L_f \sim 10$ cm showing, how large the formation zone may be in some cases.

For the boundary between plasma and vacuum [i.e. in case eqs. (49) and (39) are used for $E \gg mc^2$] the total energy of backward radiation (in vacuum) is equal to

$$W_1 = \int_0^\infty W_1(\omega)\,d\omega = \frac{32}{15}\frac{e^2}{\pi c}\omega_{p,t}\ln\frac{E}{mc^2}, \tag{53}$$

and in the region of plasma transparency (i.e. for frequencies $\omega > \omega_{p,t}$) the energy is

$$W_1' = \int_{\omega_{p,t}}^\infty W_1(\omega)\,d\omega = \frac{2}{15}\frac{e^2}{\pi c}\omega_{p,t}\ln\frac{E}{mc^2}.$$

The causes of this increase of the energy W_1, which is slower than according to eq. (50), have already been analyzed in section 7 (see also [12]).

Even in the case of eq. (50) where one boundary is crossed, the probability of the appearance of a single transition quantum, is only on the order of $W_2/\hbar\omega_m \sim e^2/\hbar c \sim \frac{1}{137}$, according to eq. (51). That is why transition counters must have many dividing boundaries. The number of boundaries is in turn limited by the necessity to have layers (plates) with a thickness comparable with or exceeding the dimension of the formation zone L_f. Nonetheless, transition counters have their advantages and are being applied [38]. Transition counters, as well as other possible devices exploiting transition radiation and scattering of various types, may find application in experimental physics.

11. Concluding remarks

Motion of sources at a constant velocity is important for electrodynamics in continuous media and, in particular, in plasmas. New problems of this sort may arise and attract attention.

To illustrate concretely the fruitfulness of understanding the physics of transition processes, I would like to give an actual example. In 1973 a paper appeared which, within the framework of the general theory of relativity, considered a charge in the centre of mass of a binary (two identical, electrically neutral stars moving in a circle relative to their centre of mass). Such an absolutely non-moving charge emits electromagnetic waves. This result is, at first glance, unexpected. However, it becomes obvious if one knows what transition scattering is and if one bears in mind that in the general theory of relativity a gravitational field affects the electromagnetic properties of the vacuum: one can say that vacuum possesses an electric permittivity and a magnetic permeability which depend on the metric tensor g_{ik}. It is thus clear that moving stars modulate permittivity and permeability and, so to say, generate permittivity waves. As a result, transition scattering of these waves by a non-moving charge occurs, and there appears a "scattered" electromagnetic wave. The understanding of this fact enables us to treat such problems as the transformation (scattering) of a gravitational wave by a charge or, what is more realistic in astrophysics, by a magnetic dipole (a pulsar) [12].

References

[1] N. Bohr, Philos. Mag. 25 (1913) 10.
[2] N. Bohr, Philos. Mag. 30 (1915) 581.
[3] N. Bohr, Mat.-Fys. Medd. Dan. Vidensk. Selsk. 18 (1948) N8.
[4] I.M. Frank and I.E. Tamm, C.R. (Doklady) Acad. Sci. USSR, 14 (1937) 109.
[5] I.E. Tamm, J. Phys. USSR 1 (1939) 439.
[6] I.M. Frank, Usp. Fiz. Nauk. 143 (1984) 111.
[7] E. Fermi, Phys. Rev. 57 (1940) 485.
[8] V.L. Ginzburg, Theoretical Physics and Astrophysics (Pergamon Press, London, New York, 1979); a new supplemented issue of this book is to appear by VNU, The Netherlands.
[9] A. Bohr, Mat.-Fys. Medd. Dan. Vidensk. Selsk. 24 (1948) N19.
[10] L.D. Landau and E.M. Lifshitz, Electrodynamics of Continuous Media (Pergamon Press, London, New York 1984).
[11] M.L. Ter-Minakelyan, High-Energy Electromagnetic Processes in Condensed Media (Wiley, New York, 1972).
[12] V.L. Ginzburg and V.N. Tsytovich, Phys. Rep. 49 (1979) N1; Phys. Lett. 79A (1980) 16. The book by the same authors, Transition Radiation and Transition Scattering—Some Aspects of the Theory (in Russian) (Nauka, Moscow, 1984). The English translation is planned for publication by Plenum Press, New York.
[13] O. Heaviside, Electromagnetic Theory, Vol. 3 (The Electrician Publ. Co., London, 1912).
[14] L.D. Landau, J. Phys. USSR 10 (1946) 25; Collected Papers (Pergamon Press, London, New York, 1965) p. 445.
[15] V.L. Ginzburg, C.R. (Doklady) Acad. Sci. USSR 24 (1939) 131.
[16] V.L. Ginzburg, J. Phys. USSR 2 (1940) 441.
[17] V.L. Ginzburg, Sov. Phys. JETP 7 (1958) 1096.
[18] V.L. Ginzburg and I.M. Frank, Dokl. Akad. Nauk SSSR 56 (1947) 699.
[19] L.S. Bogdankevich and B.M. Bolotovsky, Zh. Exp. Teor. Phys. 32 (1957) 1421.
[20] V.N. Tsytovich, Izv. VUZ Radiofiz. 29 (1986) N5.
[21] V.L. Ginzburg and V.Ya. Eidman, Sov. Phys. JETP 8 (1959) 1055.
[22] V.L. Ginzburg, J. Phys. USSR 3 (1940) 95.
[23] V.M. Agranovich and V.L. Ginzburg, Crystal Optics with Spatial Dispersion, and Excitons (Springer-Verlag, Heidelberg, 1984).
[24] I.M. Frank, Izv. Akad. Nauk SSSR Ser. Fiz. 6 (1942) 3.
[25] I.M. Frank, in: Vavilov Memorial Volume (Acad. Sci. USSR, Moscow, 1952) p. 173; Usp. Fiz. Nauk 144 (1984) 251.
[26] V.L. Ginzburg, Radiophys. Quantum Electron. 27 (1985) 601.
[27] V.L. Ginzburg and V.N. Tsytovich, Sov. Phys. JETP 61 (1985) 48.
[28] V.L. Ginzburg, in: Vavilov Memorial Volume (Acad. Sci. USSR, Moscow, 1952) p. 193.
[29] V.M. Dubovik and L.A. Tosunyan, Fiz. Elem. Chastits At. Yadra 14 (1983) 193.
[30] V.L. Ginzburg, Izv. VUZ Radiofiz. 28 (1985) 1211.
[31] L.S. Bogdankevich, Sov. Phys. Tech. Phys. 4 (1960) 992.

[32] V.L. Ginzburg, A.A. Gorbatsevich, Yu.V. Kopaev and B.A. Volkov, Solid State Commun. 50 (1984) 339.
[33] V.L. Ginzburg and I.M. Frank, Dokl. Akad. Nauk SSSR 56 (1947) 583.
[34] I.M. Frank, Usp. Fiz. Nauk 129 (1979) 685.
[35] V.L. Ginzburg and I.M. Frank. Zh. Exp. Teor. Fiz., 16 (1946) 15.
Short version of this paper: I.M. Frank and V.L. Ginzburg, J. Phys. USSR 9 (1945) 353.
[36] K.A. Barsukov, Zh. Eksp. Teor. Fiz. 37 (1959) 1106.
G.M. Garibyan. Sov. Phys. JETP 10 (1960) 372.
[37] V.B. Berestetskii, E.M. Lifshitz and L.P. Pitaevskii, Quantum Electrodynamics (Pergamon Press, London, New York, 1982).
[38] C.W. Fabian and H.G. Fischer, Rep. Progr. Phys. 43 (1980) 1003.
K. Kleinknecht. Phys. Rep. 84 (1982) N2, 87.
[39] D.A. Kirzhnits and V.V. Losgakov, JETP Lett. 42 (1985) 226.
[40] V.L. Ginzburg and V.P. Frolov, JETP Lett. 43 (1986) 265; Phys. Lett. A (in press).

Discussion, session chairman S. Belyaev

Rubbia: You have shown us situations in which a particle can emit photons. There exist also the inverse processes, in which charged particles can absorb photons, like for example inverse Vavilov–Cherenkov radiation. This kind of phenomenon can also provide us with novel ways of accelerating particles using coherent radiation.

Ginzburg: You are correct. I simply forgot to mention this.